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THEORETICAL PERFORMANCE OF SELECTED FLUID INJECTANTS FOR THRUST VECTOR CONTROL

by

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ABSTRACT

A linearized model of fluid injection thrust vector control is developed and shown to be in good agreement with experimental data. It is used to predict performance of various fluid injectants (gas and liquid; inert and reactive) in conjunction with a hypothetical rocket. The results are discussed in detail.

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LIST OF SYMBOLS

A	Streamtube cross-sectional area.
B	$\sqrt{M^2 - 1}$.
c_p	Specific heat at constant pressure (per unit mass).
D_T	Heat of dissociation (per unit mass).
F	Lateral or side force.
g	Gravitational constant.
H	Energy term defined as $dH = \Delta H_T dw/w - [(h_{gT} - h_{go}) + U^2/2](dw_g/w) - [(h_{vT} - h_\ell) + U^2/2 - V_\ell^2/2](dw_\ell/w)$.
h	Enthalpy (per unit mass).
ΔH	Constant-pressure heat of reaction (per unit mass of injectant).
I_s	Effective specific impulse of injectant.
$I_s{}^*$	Specific impulse of vacuum-exhausted sonic jet of gaseous injectant.
I_s'	$pAkM^2/w = (\sqrt{kRT/W})/g$.
I_d	Density impulse.
k	Specific heat ratio.
M	Mach number.
p	Pressure.
Q	Heat absorbed by injectant (per unit mass) to change from initial to final state; includes heats of chemical reactions.

R	Universal gas constant.
r	Radius in cylindrical coordinates.
$\bar{r}$	Radius of streamtube containing mixed gases $\pi\bar{r}^2 = S(\infty)$.
$S(\xi)$	Cross-sectional area of body of revolution. See Fig. 2.
T	Temperature.
U	Freestream velocity (undisturbed), $U^2/2 = c_p(T_o - T) = c_pT\ (k - 1)M^2/2$.
u'	x--component of perturbation velocity.
v'	r--component of perturbation velocity.
V	Injectant velocity.
W	Molecular weight.
w	Mass flow rate.
x, y, z	Cartesian coordinates, x in direction of flow, y normal to wall.
Y	Ratio of downstream component of jet velocity to freestream velocity.
θ	Angle in cylindrical coordinates.
ν	Net mole change of gas in flow system per mole of injectant.
ξ	Axial distance along body of revolution. See Fig. 2.
ρ	Density.

$(\ )_o$	Stagnation state.
$(\ )_g$	Injected gas.
$(\ )_\ell$	Injected liquid.
$(\ )_v$	Vapor.
$(\ )_T$	At temperature T.

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THEORETICAL PERFORMANCE OF SELECTED FLUID INJECTANTS FOR THRUST VECTOR CONTROL[1]

SUMMARY

This paper presents a "linearized" model for fluid injection thrust vector control and with it predicts performance of selected fluids in combination with a hypothetical high-performance rocket. The model is based on constant-area mixing between a trace of injectant and a portion of the (ideal gas) supersonic exhaust flow. Mixing, phase changes, and chemical reactions (if any) are assumed to be instantaneous and complete. After mixing, the gases expand isentropically, until their static pressure equals that of the undisturbed supersonic flow, and displace the ambient flow accordingly. The side force, which is a function of this displacement, is computed by linear supersonic flow theory. This paper shows that results of the detailed two-dimensional analysis are approximately correct for three-dimensional flows. It also presents formulas for effective specific impulse of gases and liquids, inert or reactive.

The analysis agrees excellently with our recently reported data on inert gas injection in a conical rocket nozzle. We make a proper comparison by extrapolating the measured effective specific impulse to zero injection rate; this procedure removes nonlinear effects.

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[1]Sponsored by Special Projects Office, Bureau of Naval Weapons.

This study examines the chemical aspects of fluid injection by comparing theoretical performance of various fluids injected into a hypothetical high-performance rocket motor nozzle with a fixed injection configuration. It is assumed that exhaust gases are H_2-rich, and chemical reaction between H_2 and injectant is allowed. For volume-limited application, a large density impulse is preferred. For such an application, we conclude that:

1. Compressed gases, inert or reactive, are least desirable.
2. Dense inert liquids are superior to light inert liquids or liquids that undergo endothermic decomposition, but are inferior to liquids which undergo exothermic decomposition (monopropellants).
3. Bipropellant injection, or injection of liquids which chemically react with the rocket exhaust, show the most potential.
4. Propellant gas injection compares well with reactive liquid and bipropellant injection.

I. INTRODUCTION

Thrust vector control is one of the foremost problems facing the large solid-propellant rocket industry. In principle, several methods can be used, but because high-performance propellants produce very high temperature and multiphase flow, thrust vector control methods which keep moving parts from contact with propellant gas are preferred. At present movable nozzles and fluid injection appear most promising (1).[2] This paper deals with fluid injection analysis.

Erickson and Bell (2) have summarized many of the previous experimental and theoretical studies of thrust vector control by fluid injection. But experiments have been largely uncoordinated, and results have not been correlated with an "all-inclusive" theory — nor are they likely to be, considering the spectrum of experimental variables. Emphasis has been on gas injection, and proposed theoretical models have met with limited success. Liquid injection, complicated by atomization and vaporization, is less amenable to analysis. Since the jet-induced shock wave is strongest in the vicinity of the orifice, theories tend to rely principally on the flow structure near the orifice, with wake and wall effects neglected. Jet expansion, shock-wave-boundary-layer interaction, and pressure contours make up the framework of these models.

Our analysis is based on a simple "linear" treatment, in which a <u>trace</u> of fluid is injected and the ambient supersonic flow adjusts for this perturbation. Aerothermochemical

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[2] Numbers in parentheses indicate References at end of paper.

processes, such as vaporization, chemical reactions, and mixing, are assumed to occur instantaneously near the injection point. Constant area mixing occurs between the trace of injectant and a portion of the supersonic flow, which is considered an ideal gas. Mixing causes a pressure rise and induces a weak compression wave in the enveloping flow. The transverse component of jet momentum dissipates in the mixing process. After mixing, the gases expand isentropically until their static pressure equals that of the undisturbed supersonic flow. Expansion waves in the supersonic flow maintain pressure continuity along the dividing streamline. We have developed a "linearized" equation for effective specific impulse of injectant.

II. TWO-DIMENSIONAL ANALYSIS

Consider the model shown in Fig. 1. Integrating the pressure rise along the dividing streamline gives the lateral (side) force. According to two-dimensional linear supersonic flow theory, the pressure coefficient $\delta p/(pkM^2/2) = 2(dy/dx)_{str}/\sqrt{M^2 - 1}$, where $(dy/dx)_{str}$ is the streamline slope. The side force becomes

$$dF = \iint (\delta p)\ dxdz = (kM^2/\sqrt{M^2 - 1})\ pdA \qquad (1)$$

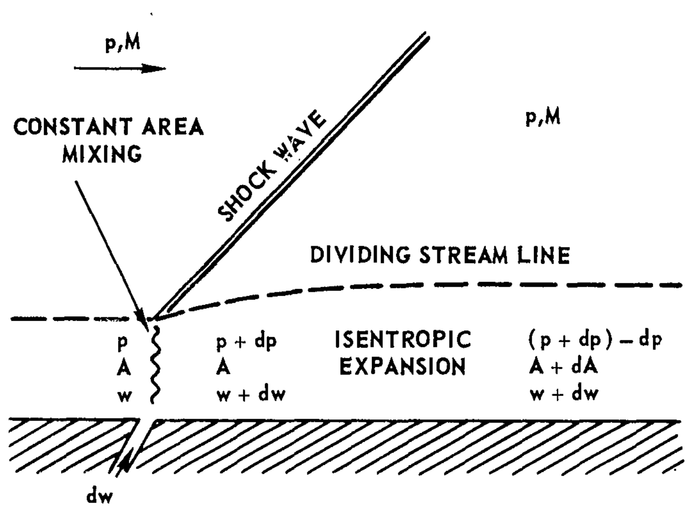

Fig. 1 "LINEARIZED" MODEL FOR FLUID INJECTION ANALYSIS (dw/w <<1)

We evaluate pdA with generalized one-dimensional flow theory, as given by Shapiro (3, Chap. 8). The pressure rise due to constant area mixing and chemical change is[3]

$$\frac{dp}{p} = \frac{kM^2}{M^2-1}\left\{\frac{dH}{c_pT} + 2\left[1 + \frac{k-1}{2}M^2\right]\frac{dw}{w} - Y\left[1 + (k-1)M^2\right]\frac{dw}{w} - \frac{dW}{W}\right\} \tag{2}$$

and, for isentropic expansion,

$$\frac{dA}{A} = \frac{M^2-1}{kM^2}\frac{dp}{p} \tag{3}$$

Combining Eqs. (1), (2), and (3), we get

$$I_s = \frac{dF}{dw} = \frac{pA}{w}\frac{kM^2}{\sqrt{M^2-1}}$$

$$\left\{\frac{w}{c_pT}\frac{dH}{dw} + 2\left(1 + \frac{k-1}{2}M^2\right) - Y\left[1 + (k-1)M^2\right] - \frac{w}{W}\frac{dW}{dw}\right\} \tag{4}$$

This "linearized" solution for the effective specific impulse is general and is reducible for special cases of interest. Specific formulas for gases and liquids, inert or reactive, are given in Table I. (Reactive injectants are fluids which chemically react with mainstream components). In these formulas, ν is the net moles change of gaseous species added to system by injection per mole of injectant; $\nu = 1$ for inert gases and vaporized inert liquids, but is

[3] Eq. (2) is not as general as given by Shapiro. We exclude heat added from external sources, external work, wall-shearing stress, and drag of stationary immersed bodies--that is, $dQ = dW_x = 4f\frac{dx}{D} = dX = 0$ in Shapiro's table of influence coefficients.

Table I

THEORETICAL FORMULAS FOR EFFECTIVE SPECIFIC IMPULSE FOR VARIOUS KINDS OF INJECTANTS

Injectant	I_s [from Eq. (4)]
Inert gas	$\frac{pA}{w} \frac{kM^2}{\sqrt{M^2-1}} \left\{ \frac{c_pT_o - (h_{gT}-h_{go})}{c_pT} + \frac{W}{W_g} + Y\,[1 + (k-1)M^2] \right\}$
Inert liquid	$\frac{pA}{w} \frac{kM^2}{\sqrt{M^2-1}} \left\{ \frac{c_pT_o - (h_{vT}-h_\ell) + V_\ell^2/2}{c_pT} + \frac{W}{W_\ell} + Y\,[1 + (k-1)M^2] \right\}$
Reactive gas	$\frac{pA}{w} \frac{kM^2}{\sqrt{M^2-1}} \left\{ \frac{c_pT - (h_{gT}-h_{go}) + \Delta H_T}{c_pT} + \nu \frac{W}{W_g} + Y\,[1 + (k-1)M^2] \right\}$
Liquid bipropellant [a]	$\frac{pA}{w} \frac{kM^2}{\sqrt{M^2-1}} \left\{ \frac{c_pT_o - \overline{(h_{vT}-h_\ell)} + \Delta H_T + \overline{V_\ell^2/2}}{c_pT} + \bar{\nu} \frac{W}{\overline{W}_\ell} + \overline{Y}\,[1 + (k-1)M^2] \right\}$

[a] $dw = dw_1 + dw_2$; $\overline{(h_{vT}-h_\ell)}\, dw = (h_{vT}-h_\ell)_1\, dw_1 + (h_{vT}-h_\ell)_2\, dw_2$; $\overline{V_\ell^2/2}\, dw$
$= (V_\ell^2/2)_1\, dw_1 + (V_\ell^2/2)_2\, dw_2$; $\overline{Y}\, dw = Y_1 dw_1 + Y_2 dw_2$; $dw/\overline{W}_\ell = dw_1/W_1 + dw_1/W_2$,
$\bar{\nu}$ is moles of products divided by moles of injectants.

usually different from unity with chemical change. The terms $(h_{gT} - h_{go})$, $(h_{vT} - h_{\ell})$, $(h_{gT} - h_{go}) - \Delta H_T$, and $(h_{vT} - h_{\ell}) - \Delta H_T$ are the enthalpy changes per unit mass of injectant for the various mechanisms, or the Q value of the reaction (positive when the injectant absorbs heat and negative when it releases heat).

The reader will recognize that $pAkM^2/w = I_s'$ is the specific impulse of optimum expanded nozzle flow with injection point expansion ratio. I_s/I_s' is approximately the "amplification factor" form frequently used by others.

Real gas effects can be included in Eqs. (2) and (3) by using the modified table of influence coefficients given by Cordullo (4).

III. THREE-DIMENSIONAL ANALYSIS

We represent the stream tube of mixed gases by a half-body of revolution as shown in Fig. 2. The basis of the model is identical to the two-dimensional case--a constant-area mixing followed by isentropic expansion. Again, we assume small area changes and use slender body theory to obtain the flow field about the body. Shapiro (3, Chap. 17) gives theoretical background. For this problem, we find it convenient

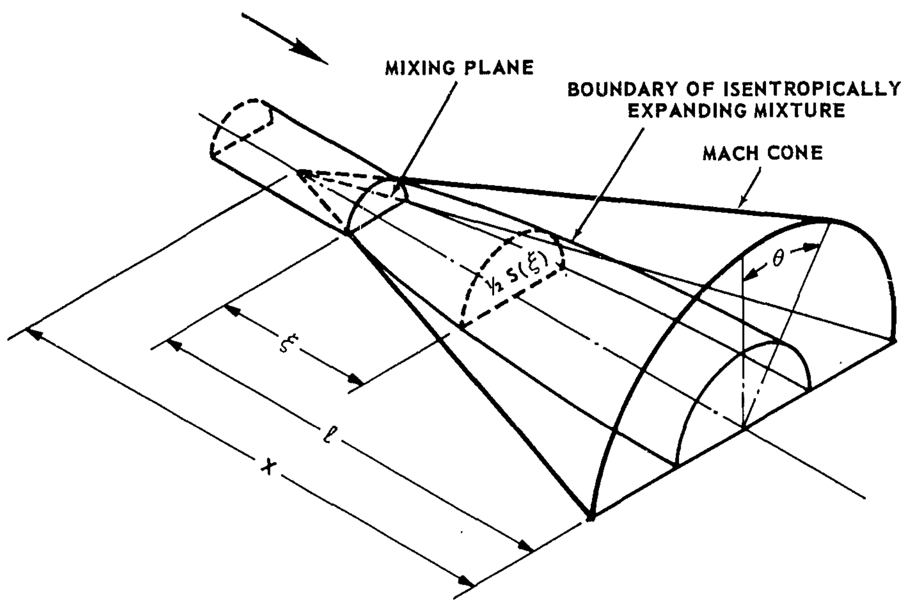

Fig. 2 THREE-DIMENSIONAL MODEL FOR FLUID INJECTION ANALYSIS

to compute side force from the y-wise stream thrust at a plane far downstream of the disturbance. Then,

$$dF = \int_{\bar{r}}^{x/B} \int_{-\pi/2}^{+\pi/2} (\rho + \rho')(U + u')\ v' \text{ Cos } \theta \ rdrd\theta$$

$$\approx 2\ \rho\ U \int_0^{x/B} v'\ rdr \qquad (5)$$

where

$$\rho + \rho' \approx \rho$$

$$U + u' \approx U$$

and

$$v'\ (x,r) = \frac{U}{2\pi} \int_0^{x-Br} \frac{d^2S}{d\xi^2}\ \frac{x-\xi}{r}\ \frac{d\xi}{\sqrt{(x-\xi)^2 - B^2r^2}}$$

according to slender body theory. $S(\xi)$ is the cross-sectional area of the body of revolution--in our case, twice the area of the stream tube containing mixed gases. Without specifying $S(\xi)$, Eq. (6) cannot be reduced further. However, it can be approximated. Since $(x - \xi)/r\sqrt{(x - \xi)^2 - B^2r^2}$ varies slowly except at the singular point $\xi = x - Br$,

$$v'(x,r) \approx \lim_{\epsilon\to 0}\left\{\frac{U}{2\pi r}\left[\left\langle\frac{x}{\sqrt{x^2-B^2r^2}}\right\rangle_{av}\int_0^{x-Br-\epsilon}\frac{d^2S}{d\xi^2}\,d\xi\right.\right.$$

$$\left.\left.+\left(\frac{d^2S}{d\xi^2}\right)\int_{x-Br-\epsilon}^{x-Br}\frac{(x-\xi)\,d\xi}{\sqrt{(x-\xi)^2-B^2r^2}}\right]\right\} \tag{7}$$

$$\approx \frac{U}{2\pi r}\sqrt{\frac{x+Br}{x-Br}}\left(\frac{dS}{d\xi}\right)_{x-Br}$$

where we place $(dS/d\xi)_o = 0$ to be consistent with slender body theory. Substituting Eq. (7) into Eq. (5) gives

$$dF \approx \frac{\rho U^2}{\pi}\int_{\bar{r}}^{x/B}\left(\frac{dS}{d\xi}\right)_{x-Br}\sqrt{\frac{x+Br}{x-Br}}\,dr \tag{8}$$

If we again employ the approximating technique used with Eq. (7), Eq. (8) reduces to

$$dF \approx (1 + 2/\pi)(kM^2/\sqrt{M^2 - 1})\ pdA \tag{9}$$

The three-dimensional solution, Eq. (9), and the two-dimensional solution, Eq. (1), are remarkably similar. They differ only by a constant. If this difference is real, slot injection would be less efficient than, say, circular orifice injection. Since we know of no experimental evidence supporting this observation, we suspect that this constant difference is a result of our approximations. It is significant that fluid properties have the same influence, regardless of orifice configuration.

IV. COMPARISON WITH EXPERIMENT

Table II compares predictions with Eq. (4) with our measurements of inert gas injection in a conical rocket nozzle (5). In this study, the propellant was a hot gas (catalytically decomposed H_2O_2). Various ambient-temperature inert gases were injected through a convergent circular orifice normal to the conical exhaust nozzle axis. Details appear in Ref. 5.

Experimentally, I_s decreases slightly with increasing flow rate. We assign this behavior to (a) nonlinear effects, (b) incomplete mixing and expansion, and (c) losses due to reflected shock waves. Recent experimental studies of these losses point to (a) and (b) as the primary causes in these experiments; these are removed by extrapolating the measured values of I_s to zero injection rate. Figure 3 is an example. Only sonic injection data are used, since they conform closely to $Y = 0$. (Unknown axial velocity components ($Y \neq 0$) are suspect for subsonic injection data.) Theory and experiment are in excellent agreement.

Table II

COMPARISON OF APL MEASUREMENTS OF INERT GAS INJECTION IN A CONICAL ROCKET NOZZLE WITH LINEAR THEORY

Propellant	Injectant [d]	I_s, sec	
		Experiment [a]	Theory
H_2O_2 [c]	H_2	490	501
	He	306	345
	0.8 He + 0.2 Ar [b]	198	191
	N_2	133	135
	Ar	130	130
	CO_2	126	122

a Obtained by extrapolating sonic data to zero injection rate.

b Mole fraction.

c $T_o = 1025°K$, $T = 580°K$, $W = 22$, $M = 2.4$, $k = 1.27$.

d $T_{oj} \approx 298°K$.

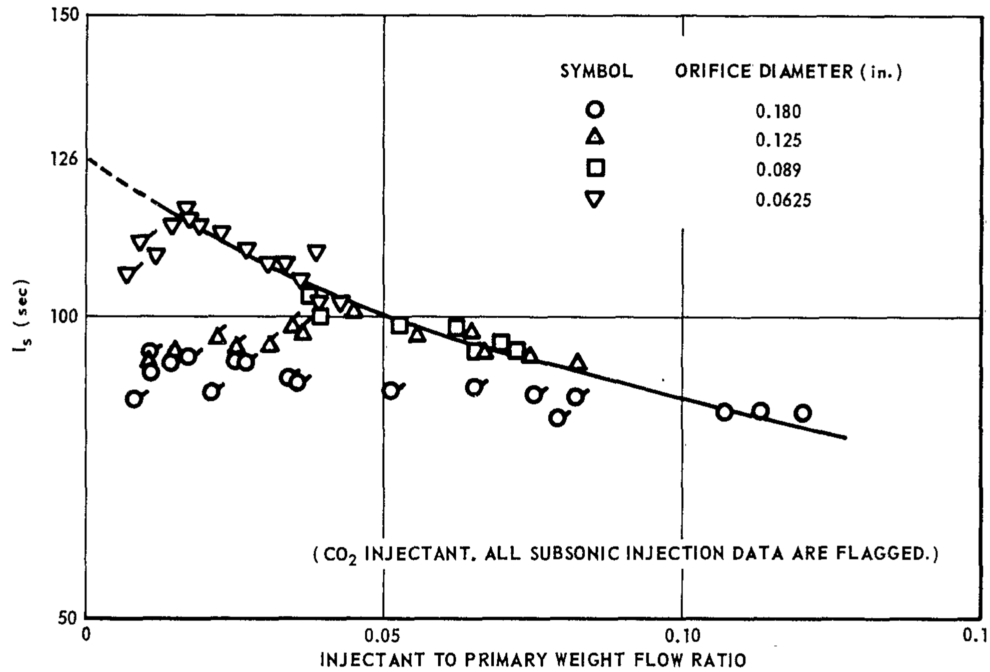

Fig. 3 EXPERIMENTAL GAS INJECTION DATA OF REF. 4, SHOWING EXTRAPOLATION TO ZERO INJECTANT FLOW RATE

V. THEORETICAL PERFORMANCE OF SELECTED FLUIDS

Many feel that better fluid injection systems will be achieved by exploiting the chemical aspects of the problem. Since linear theory provides us with a tool to judge the potential of an injectant, it seems worthwhile to compare theoretical performance of selected injectants on a common basis.

To do this, we select a hypothetical high-performance rocket motor ($k = 1.24$, $W = 22$, $T_o = 3000°K$)[4] and injection configuration ($Y = 0$, $M = 2.5$, $T = 1714°K$). We assume that exhaust gases contain excess H_2, and that chemical reaction between injectant and H_2 can occur in some cases. In practice, a multitude of chemical reactions may occur. The kinetic energy of the liquid jet is based upon no-loss injection from 1000 psi storage to 14.7 psi; it is always quite small compared to thermochemical energy. (For example, $V_\ell^2/2 = 1.65$ cal/gm for H_2O). We assume that gases are stored at 1500 psi. Without kinetic considerations, it is impossible to define a priori the degree or extent of vaporization, dissociation, or chemical reaction. We have, therefore, tended toward equilibrium thermochemistry, but in some cases more than one mechanism is considered for the same injectant (NH_3, Br_2, N_2O_4, and H_2O_2).

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[4] In our calculations, it is not necessary to specify the pressure. The motor gases are assumed to be ideal gases with the specified conditions (frozen flow), and the chemistry associated with the injectant is assigned. To be more exact, one should include pressure-dependent chemical equilibrium (dissociation), but we exclude this refinement. The propellant selected for this study has a specific impulse (1000 - 14.7) of about 260 sec.

Table III gives results for a variety of injectants in six classes: (1) inert gases, (2) inert liquids, (3) reactive gases (gases which react with H_2-rich propellant exhaust), (4) dissociative liquids, (5) reactive liquids, and (6) liquid bipropellant systems. The mechanism is expressed by a chemical equation which contains the Q value. ν values (net moles of gas added to flow system per mole of injectant) are also tabulated. For volume-limited application, the tabulated density impulse I_d is useful. Of course, we must also consider the necessary hardware associated with the injection system in any particular application.

Inert gas injectants other than propellant gases generally have I_s values which decrease with increasing molecular weight. H_2 and He do have good I_s's, but very poor I_d's. For gases of practical value, the I_d is less than about 100, although this value might be increased slightly by higher-pressure storage (1500 psi was assumed for these studies). For reactive gases, I_s values are higher, but not enough to offset the poor packaging of gaseous injectants. Gaseous injectants may be preferred where a "ready" simple fluid injection system is required.

Propellant gas injection is exceptionally good for volume-limited application (I_d = 579). Storage of condensed phase reactants is the obvious reason for its exceptional qualities. Similarly, hot gases derived from other condensed systems have large I_d's. Here we draw the reader's attention to the equivalent bipropellant systems[5] (I_d = 589 for N_2H_4-ClF_3 and I_d = 491 for N_2O_4-N_2H_4) and the somewhat poorer monopropellants (I_d = 362 for H_2O_2 and I_d = 304 for N_2H_4). These "hot gas" injectants could well represent the most efficient

[5] Not optimized for mixture ratio.

Table III

THEORETICAL EFFECTIVE SPECIFIC IMPULSE FOR VARIOUS INJECTANTS IN CONJUNCTION WITH A HYPOTHETICAL ROCKET MOTOR AND NOZZLE

Mechanism [a]	ν [b]	I_s (sec)	I_d (gm sec cm^{-3})
Inert Gases			
H_2 (g, 298) + 10.34 = H_2 (g, 1714)	1	624	4.85 [c]
He (g, 298) + 7.03 = He (g, 1714)	1	505	8.25 [c]
N_2 (g, 298) + 10.98 = N_2 (g, 1714)	1	204	23.3 [c]
O_2 (g, 298) + 11.59 = O_2 (g, 1714)	1	198	27.2 [c]
Ar (g, 298) + 7.03 = Ar (g, 1714)	1	208	35.7 [c]
Xe (g, 298) + 7.03 = Xe (g, 1714)	1	184	97.8 [c]
Ra (g, 298 + 7.03 = Ra (g, 1714)	1	181	164. [c]
Propellant Gases	1	349	579. [d]
Inert Liquids			
H_2O (ℓ, 298) + 24.5 = H_2O (g, 1714)	1	127	127
CCl_2F_2 (ℓ, 298) + 37.5 = CCl_2F_2 (g, 1714); Freon 12	1	159	205
$CClF_2CClF_2$ (ℓ, 298) + 43.9 = $CClF_2CClF_2$ (g, 1714); Freon 114	1	155	223
CO_2 (ℓ, 298) + 18.8 = CO_2 (g, 1714)	1	171	188
NH_3 (ℓ, 298) + 13.1 = NH_3 (g, 1714)	1	208	170
Br_2 (ℓ, 298) + 20.0 = Br_2 (g, 1714)	1	173	539
Hg (ℓ, 298) + 21.7 = Hg (g, 1714)	1	172	2330
O_2 (ℓ, 90)[e] + 14.6 = O_2 (g, 1714)	1	186	212
H_2 (ℓ, 14)[e] + 12.4 = H_2 (g, 1714)	1	491	34
N_2 (ℓ, 77)[e] + 13.9 = N_2 (g, 1714)	1	192	155
Reactive Gases [g]			
O_2 (g, 298) + $2H_2$ (g, 1714) = $2H_2O$ (g, 1714) + 102.3	0	642	88 [c]
Dissociative Liquids			
NH_3 (ℓ, 298) + 37.1 = 1/2 N_2 (g, 1714) + 1-1/2 H_2 (g, 1714)	2	161	132
N_2O_4 (ℓ, 298 + 50.3 = $2NO_2$ (g, 1714)	2	154	223
N_2O_4 (ℓ, 298) + 77.5 = 2NO (g, 1714) + O_2 (g, 1714)	3	141	204
H_2O_2 (ℓ, 298) + 6.79 = H_2O (g, 1714) + 1/2 O_2 (g, 1714)	1.5	247	362
N_2H_4 (ℓ, 298) + 19.7 = N_2 (g, 1714) + 2 H_2 (g, 1714)	3	304	304
Fe $(CO)_5$ (ℓ, 298) + 124 = Fe (s, 1714) + 5 CO (g, 1714)	5	163	237

Table III (Cont'd)

THEORETICAL EFFECTIVE SPECIFIC IMPULSE FOR VARIOUS INJECTANTS IN CONJUNCTION WITH A HYPOTHETICAL ROCKET MOTOR AND NOZZLE

Mechanism [a]	ν [b]	I_s (sec)	I_d (gm sec cm^{-3})
Reactive Liquids [g]			
N_2O_4 (ℓ, 298) + $4H_2$ (g, 1714) = N_2 (g, 1714) + $4H_2O$ (g, 1714) + 206.1	1	473	692
H_2O_2 (ℓ, 298) + H_2 (g, 1714) = $2H_2O$ (g, 1714) + 53.2	1	434	635
ClF_3 (ℓ, 298) + $2H_2$ (g, 1714) = HCl (g, 1714) + 3HF (g, 1714) + 148.3	2	424	767
Br_2 (ℓ, 298) + H_2 (g, 1714) = 2HBr (g, 1714) + 5.97	1	193	602
O_2 (ℓ, 90)[e] + $2H_2$ (g, 1714) = $2H_2O$ (g, 1714) + 105.3	0	586	669
[$NaClO_4$ + 6.8 H_2O] (soln., 298)[f] + 4-1/2 H_2 (g, 1714) + 54.4 = HCl (g, 1714) + Na (g, 1714) + 10.8 H_2O (g, 1714)	1.07	222	---
Bipropellant			
N_2H_4 (ℓ, 298) + ClF_3 (ℓ, 298) = N_2 (g, 1714) + HCl (g, 1714) + 3HF (g, 1714) + 129.4	2.5	393	589
$2N_2H_4$ (ℓ, 298) + N_2O_4 (ℓ, 298) = $3N_2$ (g, 1714) + $4H_2$ (g, 1714) + 166.5	2.33	406	498

a Number in equation is kcal/mole of injectant; number in parentheses is temperature in degrees Kelvin, ℓ = liquid, g = gas, s = solid.
b Moles change of gas in flow system per mole of injectant.
c Gas density based on storage at 100 atm and 298°K.
d Solid propellant density = 1.66 gm/cm^3.
e Cryogenic.
f $NaClO_4$ - H_2O solution, 50% by weight of each.
g H_2 (g, 1714) supplied by rocket exhaust.

practical fluid injectant systems for thrust vector control. Remarkably enough, they are not "potentially" the best.

We find a broad span in I_d values in the inert liquids category. I_s values are usually modest, ranging from 100 to 200 sec. (Liquid H_2, with its small molecular weight, is exceptional with $I_s = 491$, but again its I_d is poor.) Examples of inert liquids with high density impulses are Br_2 and Hg. In addition Br_2 has the extra potential of reacting with H_2; this slightly improves its performance. Some heavy liquids used in mineralogical analyses may have virtue as fluid injectants; for example, acetylene tetrabromide, thallous formate water solutions, and stannic bromide with carbon tetrachloride, all have densities near 3 gm/cm^3. (We could not locate thermochemical data on these compounds; hence they are not included in Table II.) Heavy liquids have a distinct advantage over the seemingly-superior reactive liquids: Their performance depends only on heat transfer for vaporization, and does not rely on chemical reactions.

The reactive liquids--oxidizers which may react with H_2 in the mainstream--are potentially best. Some of these are considered storable--(ClF_3, N_2O_4)--an obvious advantage for missile application. Of the examples considered, ClF_3 is best ($I_d = 767$). One example, a salt solution 50% by weight of $NaClO_4$ in H_2O, is a type of liquid injectant (solution or mixture) that could well be explored further, but thermochemical and related data for such injectants are usually lacking. For example, we could not find the heat of solution (assumed zero for the calculations) or the solution density. But both are easily determined in the Laboratory. The practical performance of reactive injectants, which depends strongly on mixing and chemical kinetics, can only be answered by experiment.

Another important observation from Table III needs emphasizing. We found that endothermic decomposition always decreases performance. Note especially the better performance for incomplete dissociation of N_2O_4 ($N_2O_4 = 2NO_2$ as compared to $N_2O_4 = 2NO + O_2$) and for nondissociated NH_3, compared to dissociated NH_3. Even $Fe(CO)_5$, which speculatively gives five moles of gas per mole of injectant, is average. Although dissociation provides additional gas volume, the heat absorbed usually offsets this gain. A mathematical basis for this observation is found in the theory.

The difference between I_s with dissociation and I_s without dissociation is

$$\Delta I_s = \frac{pA}{w} \frac{kM^2}{\sqrt{M^2-1}} \left[- \frac{D_T}{c_p T} + (\nu - 1) \frac{W}{W_L} \right]$$

where D_T is the dissociation energy. It follows that I_s decreases with dissociation if

$$(\nu - 1) < \frac{W_L \, D_T}{W \, c_p T}$$

where $W_L \, D_T$ is the molar heat of dissociation and $W \, c_p T$ is the molar heat content of the mainstream. $W \, c_p T \approx 18$ kcal/mole for our example and is probably typical. If only a single chemical bond is broken, $W_L \, D_T$ will be the bond energy and $\nu = 2$. Since typical bond energies exceed 18 kcal/mole, the inequality suggests that performance loss occurs with dissociation. Increasing T by injecting nearer the nozzle throat will reduce this loss.

Secondary-injection-induced nucleation, recombination of mainstream species, or other thermochemical processes are tractable with the linearized solution. These items are usually overlooked entirely in other analyses.

Obviously, freestream conditions and injection angle, which were stationary for the Table III calculations, affect performance. Figures 4A through 4D show the effects of these variables for typical systems with:

A. Inert gas (sonic injection).

B. Inert liquid.

C. Dissociative liquid.

D. Reactive liquid.

Rocket motor conditions are the same as before. All curves are similar in contour, with a minimum in I_s at $M \approx 2$ and a singularity at $M = 1$. The singularity is usual in linear supersonic flow theory, and has no practical value. I_s increases monotonically with increasing $M > 2$. $M > 5$ are seldom encountered in practice.

All practical injection angles are bounded between the upstream and downstream injection curves. The theoretical effect of injection angle is small for liquid injection; the liquid jet momentum is just too small to have much effect. [$\left|Y\right|_{max} \approx 0.05$, $H_2O(\ell)$ @ $M = 2.5$]. Experimentally, it is sometimes found that upstream injection of liquids leads to better performance; this is probably because longer residence time aids vaporization and mixing. Injection angle effects are more pronounced with gaseous injectants where the jet momentum is larger [$\left|Y\right|_{max} \approx 0.16$, $N_2(g,298)$ @ $M = 2.5$] . If the gaseous injectant is heated, as for propellant gas injection, injection angle effects will be even more pronounced.

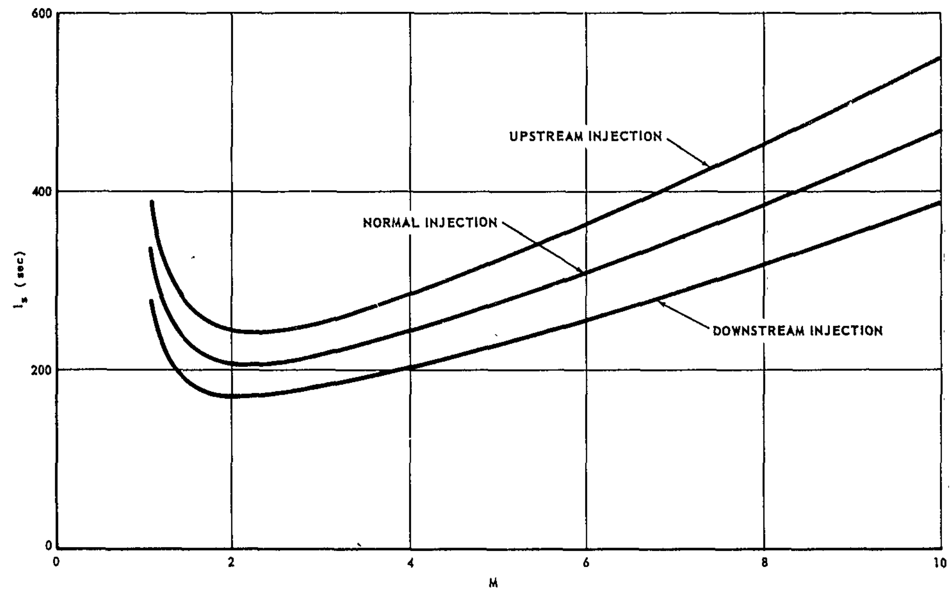

Fig. 4A INJECTION ANGLE AND MACH NUMBER EFFECTS, INERT GAS SYSTEM: $N_2 (g, 298) + Q = N_2 (g, T)$

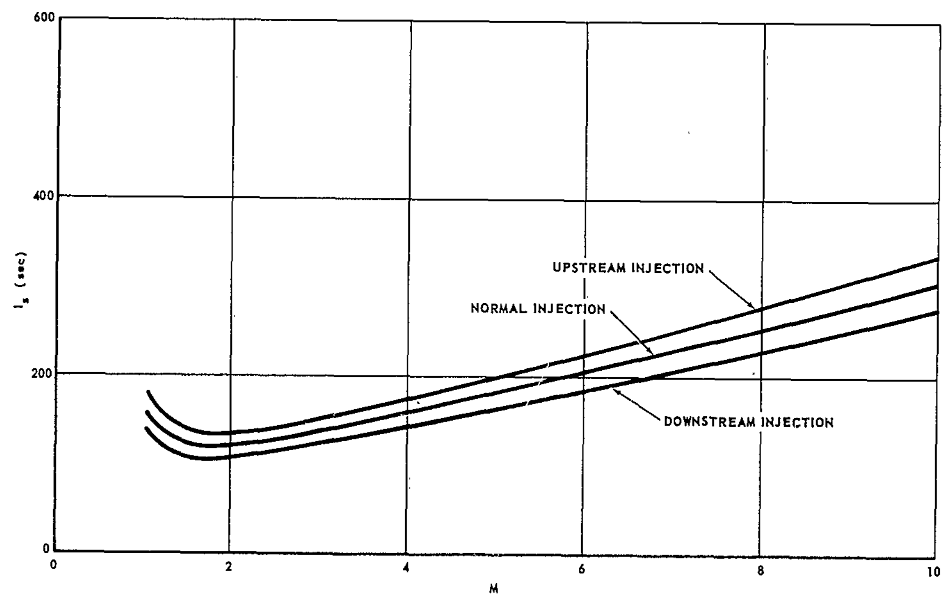

Fig. 4B INJECTION ANGLE AND MACH NUMBER EFFECTS, INERT LIQUID SYSTEM: $H_2O (\ell, 298) + Q = H_2O (g, T)$

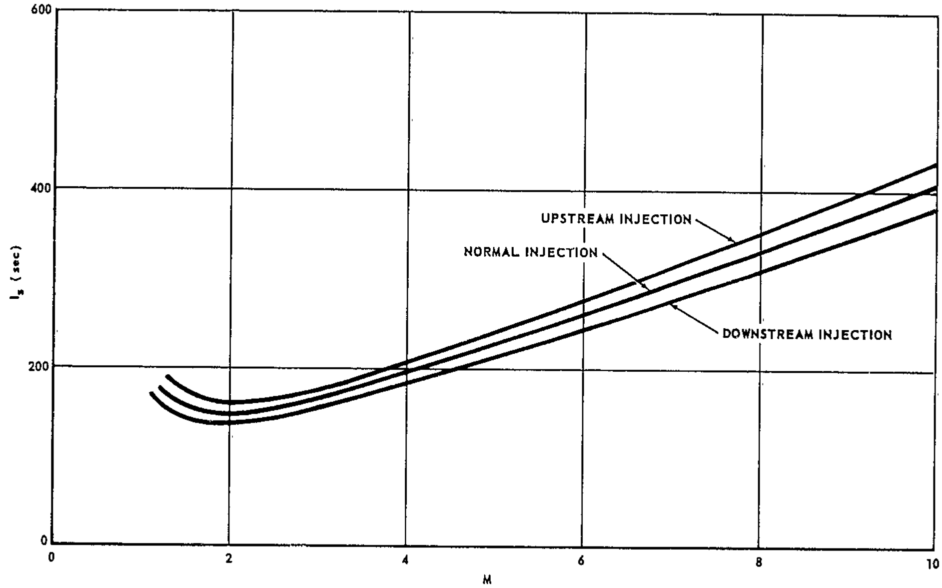

Fig. 4C INJECTION ANGLE AND MACH NUMBER EFFECTS, DISSOCIATIVE LIQUID SYSTEM: N_2O_4 (ℓ, 298) + Q = $2NO_2$ (g, T)

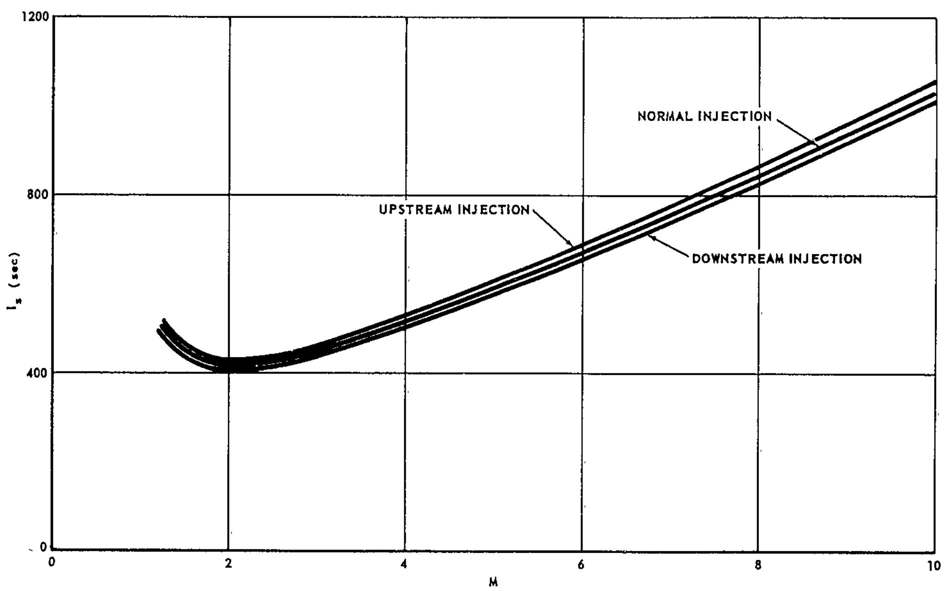

Fig. 4D INJECTION ANGLE AND MACH NUMBER EFFECTS, REACTIVE LIQUID SYSTEM: ClF_3 (ℓ, 298) + H_2 (g, T) + Q = HCl (g, T) + 3HF (g, T)

VI. A BOUND FOR NONLINEAR MIXING EFFECT

Mixing and shock-wave losses are not considered in the linearized solution. We can crudely estimate possible nonlinear mixing loss on I_s by assuming constant area mixing with a finite mixing rate; a lower "bound" is found for a mixing rate sufficient to give "choked" flow aft of the mixing plane. Mathematically, this is the maximum rate of mixing. Subsequent calculations remain the same as the linear analysis--isentropic expansion of mixed gases and isentropic deflections of ambient supersonic flow (i.e., we assume Eq. (1) is valid).

For an example, we select identical injectant and mainstream gases with injection perpendicular to the wall (Y = O) with k = 1.4. Figure 5 shows the results as an amplification

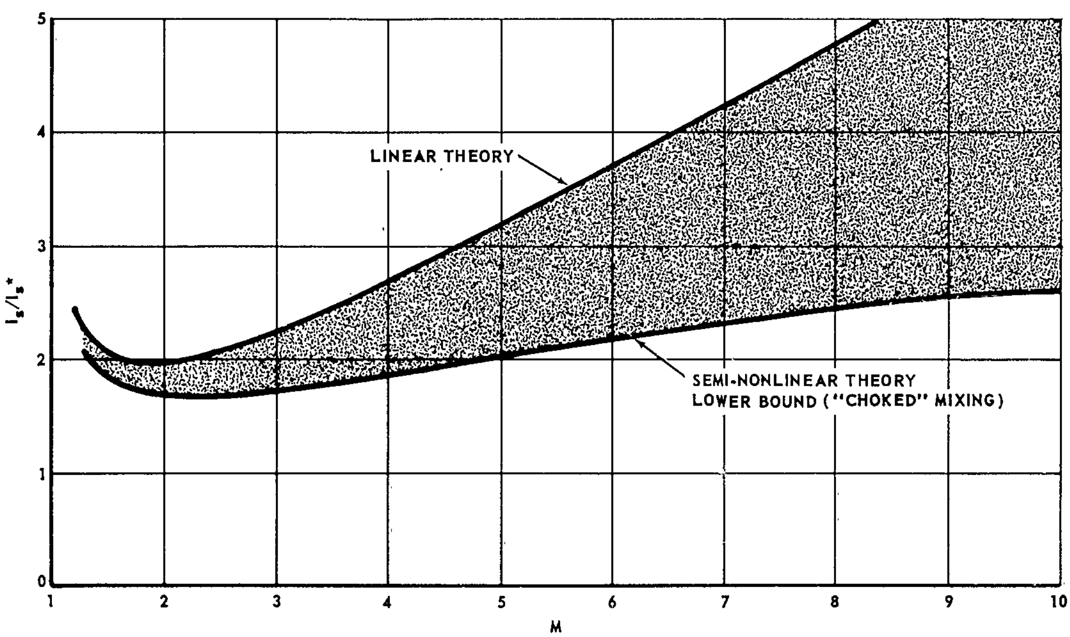

Fig. 5 NONLINEAR EFFECTS (γ = 1.4 and Y = 0)

factor I_s/I_s* versus M. I_s* is the specific impulse of a sonic vacuum-exhausted jet. The semi-nonlinear solution,[6] with the lower bound for choked mixing, always gives I_s values below linear theory. The difference increases with increasing Mach number. For Mach numbers encountered in practice ($M \approx 4$ or less), the difference between linear and semi-nonlinear theory is less than 30 per cent. Thus, the linearized solution is meaningful only for very low injection rates. But it remains a useful tool to judge the potential of a fluid injectant.

[6] Semi-nonlinear, since shock wave losses have been neglected.

VII. CONCLUSIONS

1. This study shows that a linearized model for thrust vector control by fluid injection can:

   a. Provide a very simple method of computing potential effective specific impulse for <u>any</u> fluid injectant and rocket motor combination and geometry.

   b. Be in exceptional agreement with our recently reported data on inert gas injection in a conical rocket nozzle.

   c. Predict slightly optimistic performance when compared with a model giving a lower bound for nonlinear mixing loss.

   d. Permit estimates of secondary effects (nucleation, recombination, etc.) on effective specific impulse.

2. Theoretical calculations of effective density impulse of selected fluid injectants lead to the following conclusions with respect to application:

   a. Compressed gases, inert or reactive, are least desirable.

   b. Dense inert liquids are superior to light inert liquids or to liquids which undergo endothermic decomposition, but are inferior to liquids which undergo exothermic decomposition. Some dense inert liquids such as Br_2 and Hg have exceptional theoretical merit.

c. Bipropellant injection or injection of liquids which chemically react with the rocket exhaust show the greatest potential.

d. Propellant gas injection (hot gas from the rocket chamber) compares well with reactive liquids and bipropellants.

e. All injectants behave similarly to variations in freestream Mach number. Effective specific impulse is a minimum at $M \approx 2$ and increases monotonically with $M > 2$.

f. Injection angle effects are almost insignificant for liquids but important for gases.

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